

Effect of Electrode Density and Measurement Noise on the Spatial Resolution of Cortical Potential Distribution

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Abstract—The purpose of the present study was to examine the spatial resolution of electroencephalography (EEG) by means of inverse cortical EEG solution. The main interest was to study how the number of measurement electrodes and the amount of measurement noise affects the spatial resolution. A three-layer spherical head model was used to obtain the source-field relationship of cortical potentials and scalp EEG field. Singular value decomposition was used to evaluate the spatial resolution with various measurement noise estimates. The results suggest that as the measurement noise increases the advantage of dense electrode systems is decreased. With low realistic measurement noise, a more accurate inverse cortical potential distribution can be obtained with an electrode system where the distance between two electrodes is as small as 16 mm, corresponding to as many as 256 measurement electrodes. In clinical measurement environments, it is always beneficial to have at least 64 measurement electrodes.

Index Terms—Cortical potential distribution, electroencephalography (EEG), inverse problem, spatial resolution, spherical head model, singular value decomposition (SVD).

I. INTRODUCTION

THE PURPOSE of the present study was to examine the spatial resolution of electroencephalography (EEG) by means of inverse EEG solution. The objective was to evaluate the relationship between the number of EEG electrodes, measurement noise, and the accuracy of the inverse cortical potential distribution.

EEG is a traditional noninvasive means of studying the function of the brain. EEG is characterized by its good temporal resolution. The spatial resolution of EEG is affected by blurring caused by volume conductor effects. The contribution of the low-conducting skull is particularly significant [1]. If the spatial resolution is poor, localization of complex electrical activation is difficult. Accurate source localization is important in research on evoked potentials and spontaneous brain activity. Also, in planning tumor and epilepsy surgery, precise localization of the areas causing symptoms is of importance. The spatial

resolution of the traditional 10–20-electrode system is not sufficient for modern brain research. To achieve improvement, the number of measurement electrodes needs to be increased and some sort of spatial enhancement method should be applied to the signal.

One of the most commonly applied spatial enhancement methods is to solve the cortical potential distribution, which gives an improved idea of the locations of electrical sources within the brain. Some of the methods are based on application of the Laplacian operator to the scalp recorded EEG potentials. The earliest approaches approximated the generators locally [2]. So-called global Laplacian methods have since been developed by many research groups. In global methods, Laplacian estimation is based on the potentials at all electrode locations.

Also, methods based on actual head geometry to improve the scalp surface model have been studied. Le *et al.* [3] used the measured electrode locations to construct a model for the scalp surface and Babiloni *et al.* [4], [5] used magnetic resonance images (MRIs) to construct a realistically shaped model of the scalp surface and to take into account the scalp thickness. Common to the Laplacian methods is that the potential fields are not calculated on the basis of the actual resistivities of the tissues and the geometry of the head; the current sources and sinks are approximated with the Laplacian operator.

Methods approximating the cortical potential distribution with a volume conductor model of the head have been developed. Keartfott, Sidman and colleagues [6], [7] have developed a cortical imaging technique where the head is represented by a homogeneous spherical model. A dipole layer is placed inside the cortical surface and an inward harmonic continuation problem is solved. Later, Wang and He [8] further developed this method by applying a slightly modified Rush and Driscoll three-layer spherical head model [9]. He *et al.* [10] have also developed a method where a boundary element model (BEM) of the head is constructed. In this method, the transfer matrix is formed according to boundary elements on the cortical surface, the geometry of the model, and the ratio between the conductivity of the scalp and the skull. The cortical potential field is estimated by taking the general inverse of the transfer matrix.

Groups under Gevins [11] and Le [12] have also developed a model-based method which they call deblurring. In this approach, the cortical potential distribution is determined by calculating forward solutions of an estimated cortical potential distribution and finding the one which best matches the actual measured scalp EEG.

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In all previously described methods, which estimate cortical potential distribution from the measured scalp EEG potentials, the main purpose has been to construct the cortical potential distribution and then in some way to validate its accuracy. One frequently applied validation method has been to place a few known source dipoles inside the head and to compare the resulting calculated cortical potential distribution and the approximated cortical potential distribution [6], [8], [10]. Another possibility is to compare the approximated cortical potential distribution to invasively measured actual cortical potentials [12].

Some research has been conducted where the accuracy of the EEG inverse solution or the spatial resolution of EEG has been studied as a function of the distance between EEG electrodes. Malmivuo and Suihko [13], [14] studied the spatial resolution of EEG in a noiseless situation. They conducted their studies with the concept of half-sensitivity volume (HSV).

Laarne *et al.* [15] compared the dipole localization accuracy of a 10–20-electrode system and a 64-electrode system, with different resistivity values for the skull and with noise estimate. Liu *et al.* [16] studied the source localization accuracy of EEG montages of 30 and 61 electrodes in the presence of noise. The source was modeled with a current density distribution within the cortex. Lantz *et al.* [17] studied how many electrodes are needed to localize epileptic sources. They studied the number of electrodes from clinically measured EEG using 123 electrodes and the subsets of 31 and 63 electrodes. They also studied the number of electrodes from simulations where the number of electrodes was varied between 25 and 166. In all of these studies, the benefits of increasing the number of electrodes was observed.

In the present study, the purpose was to use a model-based cortical potential estimation method to study factors affecting the accuracy of the cortical potential distribution and to identify the factors which limit the best possible spatial resolution. Today, EEG systems of even 256 electrodes exist but there are no studies about their benefits compared to sparser electrode systems. In the present study, we also examine these very dense electrode systems. We study different electrode configurations with different amounts of measurement noise with an eye to estimating the maximum spatial resolution obtained with these electrode configurations and approximating the maximum number of scalp EEG electrodes which still add accuracy to inverse cortical potential distribution.

In the present study, we apply the term spatial resolution to define how accurately an equivalent cortical source can be identified. We can expect that, as the measurement noise increases, the potential distribution of the equivalent source on the cortical surface is more smeared and, thus, the spatial resolution decreases. Our purpose was not to define specific values for spatial resolution but to examine its behavior as the number of electrodes and measurement noise is varied.

II. MATERIAL AND METHODS

In the forward problem of EEG, the source and the volume conductor are known but the measured field is unknown [18]. The forward problem can be described with the equation

$$\mathbf{b} = \mathbf{A}\mathbf{x} \quad (1)$$

where \mathbf{b} is a vector containing information on the measured field, \mathbf{x} is a vector containing information on the source, and \mathbf{A} is the forward transfer matrix containing the information on the volume conductor.

In the inverse problem, the purpose is to solve \mathbf{x} . The inverse problem does not have a unique solution. To solve the inverse problem, the degrees of freedom of the source have to be constrained. In this study, we choose to study the cortical potential distribution as the equivalent source. It can be considered as an equivalent source, because it is the electric field produced by all of the electrical sources within the brain. To solve the inverse problem, the matrix \mathbf{A} needs to be inverted. Because the system is ill-posed, in the method developed here, truncated singular value decomposition (TSVD) is applied to study the inversion of the matrix \mathbf{A} .

A. Volume Conductor Model and Forward Solution

In the present study, the head was modeled with a three-layer spherical Rush and Driscoll head model including the layers of scalp, skull, and gray matter [9]. The radii of the spheres were 92, 85, and 80 mm, respectively. The resistivity ratio between the tissues (1:15:1) was different from that of the Rush and Driscoll model [19].

To obtain the forward transfer matrix, we constructed a finite difference model (FDM) of the head. At the Ragnar Granit Institute, an FDM software has been developed [20]. This software was used to solve the FDM of the spherical head, though analytical methods could also have been used. In [21], it was proven that this FDM method works correctly compared to analytical model. The FDM is also advantageous because in our future studies the model can be easily constructed from segmented magnetic resonance images [20].

From the FDM constructed, the cortical surface was so defined that it formed a closed surface. Thus, the volume inside this surface could be omitted based on Gauss's law. Because we used the spherical head model, the cortical surface was also modeled with a sphere. The cortical surface consisted of 67 640 nodes in the FDM resistor network. To decrease the computational load of the forward transfer matrix, groups of adjacent nodes were combined to form source areas on the cortical surface. In this manner, the cortical surface was divided into equal-sized source areas. The potentials of these areas formed the vector \mathbf{x} in (1). Because the purpose was to study the spatial resolution of EEG, we studied different numbers of source areas. The numbers varied here between 52 and 8450 corresponding to the average source area size between 1547 and 9.5 mm².

We studied the five different electrode systems listed in Table I. The distances between electrodes were 60, 33, 23, 16, and 11 mm. These are the distances in the commonly applied 10–20-system [11] and the systems of 64 [11], 128 [11], 256 [22], and 512 electrodes, respectively. In our study, we placed the electrodes evenly over the entire spherical scalp surface according to the electrode distance. This is illustrated in Fig. 1. The resulting numbers of electrodes in the model were 34, 104, 232, 462, and 938, corresponding to the previously mentioned realistic systems. The potentials at the electrode locations formed the vector \mathbf{b} which is known in (1). Spherical coordinates for each electrode position were specified based

TABLE I
ELECTRODE SYSTEMS STUDIED. IN THE FIRST COLUMN, THE INTERELECTRODE DISTANCE IS GIVEN AND IN THE SECOND THE REFERENCE FOR THE DISTANCE. IN THE THIRD COLUMN, THE CORRESPONDING NUMBER OF ELECTRODES IN THE REALISTIC ELECTRODE SYSTEMS IS GIVEN AND IN THE FOURTH THE NUMBER OF ELECTRODES PLACED ON THE ENTIRE SURFACE OF THE SPHERICAL MODEL IS GIVEN

Electrode distance (mm)	Reference	Realistic electrode system	Electrode system on the spherical surface
60	[11]	21 electrodes	34 electrodes
33	[11]	64 electrodes	104 electrodes
23	[11]	128 electrodes	232 electrodes
16	[22]	256 electrodes	462 electrodes
11	approximated	512 electrodes	938 electrodes

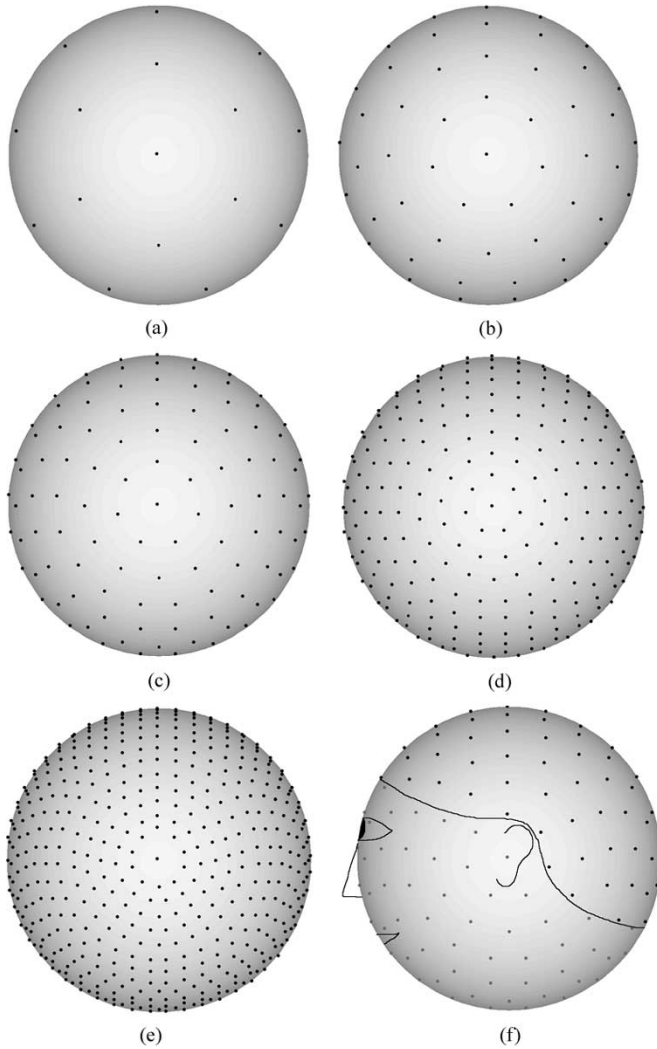


Fig. 1. Placement of the electrodes. The studied electrode systems are illustrated in subfigures (a)–(e). The interelectrode distance is (a) 60 mm, (b) 33 mm, (c) 23 mm, (d) 16 mm, and (e) 11 mm. Electrodes are placed as equidistant as possible over the spherical surface. (f) The number of electrodes used in simulations. The black electrodes are those in the realistic electrode system. The black and gray electrodes placed evenly on the entire spherical surface are those applied in our simulations.

on the distance between adjacent electrodes. According to the spherical coordinates, the point electrodes were placed on the nodes of the scalp surface in the FDM resistor network. The electrodes were placed as equidistant as possible within the

limits posed by the discrete model. By placing the electrodes evenly on the entire spherical scalp surface, we were conveniently able to study how dense the electrodes should be placed on the scalp to obtain the underlying cortical potentials as accurately as possible.

We obtained the forward transfer matrices with the FDM solver. The potential on each electrode location was found by computing the electric field generated by each source area one by one. The placement of the electrodes evenly on the entire scalp surface allows the use of full spherical symmetry to reduce the number of calculations. The results were combined into a forward transfer matrix [23].

B. Singular Value Decomposition

The accuracy of the inverse EEG solution was studied with singular value decomposition (SVD). In addition to being a convenient method to handle the ill-posedness of the forward transfer matrix, SVD is a practical means of studying the effect of measurement noise on the spatial resolution. In SVD, the forward transfer matrix A is decomposed into a product of three matrices as follows:

$$A = U\Sigma V^T \quad (2)$$

where U is an $m \times m$ orthogonal matrix, V is an $n \times n$ orthogonal matrix, and Σ is an $m \times n$ diagonal matrix whose elements σ_i are real and nonnegative. The column vectors v_i are basis vectors of the source space and the columns u_i are the basis vectors of the surface potential space [24]. The singular values σ_i of the matrix A are organized in nondescending order: $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_n \geq 0$. [25].

We adopted the method to study the effect of measurement noise from Schneider *et al.* [24], who applied it to electrocardiography (ECG). In this method, the nullspace of the matrix A consists of the basis vectors which lead to the signal smaller than the measurement noise

$$\sigma_i / \sigma_1 < \|e\| / \|b\| \quad (3)$$

where e is the measurement error in b , including noise. The relation in (3) is known as relative noise level (NL). The null space starts with the index i of the first normalized singular value smaller than the relative noise level. The basis vectors v_i belonging to the null space cannot be reconstructed. As the relative noise level increases, the number of reconstructable basis vectors decreases. This results in a more smeared inverse solution of the source potential distribution on the cortical surface, i.e., the spatial resolution decreases.

Generally, if the errors in b are much larger than errors in matrix A , the null space of A can be approximated as in (3) [26]. In the case of modeling the human body, the errors in the measurement may not be substantially smaller than the errors in the model. However, in this study, we took into account only the effect of measurement noise in the inverse EEG problem.

Schneider *et al.* and Dössel *et al.* have used for ECG relative noise levels 0.005 [24] and 0.001 [27]. Wang *et al.* have used for

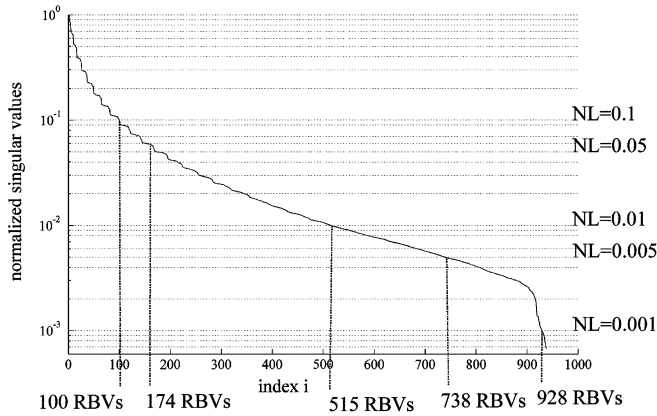


Fig. 2. Behavior of the normalized singular values. In this example, the number of electrodes on the spherical surface is 938 and the number of cortical surface areas is 2114. The number of reconstructable basis vectors (RBVs) is marked for NLs 0.001, 0.005, 0.01, 0.05, and 0.1. The SVD components that cannot be reconstructed are dominated by the measurement noise because the noise components are amplified in the inverse solution as the ratio $1/\sigma_i$ becomes large.

EEG relative noise levels between 0.005 and 0.25 [8]. We applied SVD to all calculated forward transfer matrices and solved the number of reconstructable basis vectors for each transfer matrix. The number of these vectors was studied at relative noise levels of 0.001, 0.005, 0.01, 0.05, and 0.1.

III. RESULTS

The behavior of normalized singular values in (3) is sketched in Fig. 2. In the figure, the normalized singular values of a matrix corresponding to 938 electrodes on the spherical surface and 2114 cortical source areas are sketched in logarithmic scale. When the relative noise level is high, only a small portion of the basis vectors can be reconstructed. All of the other SVD components are dominated by the measurement noise [26]. These first singular components give only a rough approximation of the cortical potential distribution. As the relative noise level decreases, more and more of the singular components can be reconstructed as the domination of noise on the small singular values decreases. Even though the amplitudes of these singular values are small, inclusion of these values in the inversion gives the fine details to cortical potential distribution. Thus, the accuracy of the inverse solution of the cortical potential distribution also increases.

In the practically noiseless case ($NL = 0.001$), the spatial resolution is limited by the volume conductor effects and the number of electrodes. This is depicted in Fig. 3(a), where the number of reconstructable basis vectors is sketched for different electrode systems, when $NL = 0.001$. From this figure, it will be seen that, with all electrode systems studied, the maximum number of reconstructable basis vectors is equal to the number of electrodes placed evenly on the entire spherical surface. Thus, in all systems, the number of electrodes on the spherical surface sets the limit to the maximum number of reconstructable basis vectors and, thus, to the maximum spatial resolution which can be obtained with the inverse solution.

In Fig. 3(b)–(e), the effect of measurement noise is sketched with $NL = 0.005$, $NL = 0.01$, $NL = 0.05$, and $NL = 0.1$, re-

spectively. From these figures, the actual influence of the measurement noise can be evaluated. It can be clearly seen that, as the measurement noise increases, the advantage of dense electrode systems is reduced. If $NL = 0.005$ or 0.01, the 512 electrodes are adequate to obtain the best possible spatial resolution. With $NL = 0.05$, 128 electrodes are adequate, and, with $NL = 0.1$, 64 electrodes are adequate. Table II depicts the approximated relative noise level ranges, in which a certain electrode system is needed to obtain best possible spatial resolution.

The results are summarized in Fig. 4 for realistic electrode systems. As a conclusion, with sparse electrode systems, the actual number of electrodes limits the spatial resolution and with the relative noise levels studied, the measurement noise has no effect on the spatial resolution. On the other hand, with dense electrode systems, the measurement noise is a critical limiting factor in the inverse solution. As the noise increases, the spatial resolution decreases considerably.

IV. CONCLUSION

A. SVD

The SVD-based method studied in this project is a convenient means of studying the inverse EEG problem, in which the source is defined as the cortical potential distribution and the effect of measurement noise is to be taken into account. The method offers good possibilities to study and evaluate the spatial resolution of EEG by examining the number of reconstructable basis vectors with different amounts of measurement noise.

B. Spatial Resolution

From the results obtained, we may conclude that, with dense electrode systems, where the distance between electrodes is smaller than 23 mm (128 electrodes), measurement noise is the factor limiting spatial resolution. Within the range of studied relative noise levels, the poor spatial resolution of the 10–20 system is caused solely by the small number of electrodes. Thus, it is preferable to use more electrodes than in the 10–20 system in any situation.

Based on our results, it is evident that the advantages of high-resolution EEG devices are highly dependent on the amount of noise in the measurement. To gain advantage from 512 or 256 electrodes, the relative noise level needs to be below 0.02 or 0.04, respectively.

The results obtained regarding the increased measurement noise agree with those of a previous study conducted by Laarne *et al.* [15]. In their paper, the dipole localization accuracy was studied with a 10–20 system and with a 10–10 electrode system. It was concluded that as the noise is increased more electrodes than in the 10–20 system are needed to solve the inverse problem more accurately.

The results obtained in this study also agree with those obtained by Malmivuo and Suihko [14]. As explained above, they studied the spatial resolution of EEG with the concept of HSV. They showed that, with different tissue resistivity ratios (including 1:15 for brain:skull), the spatial resolution is improved even when the interelectrode distance is smaller than 10 mm in a noiseless case. From their study, it can also be seen that, as the distance between two electrodes is over 50 mm, the HSV does

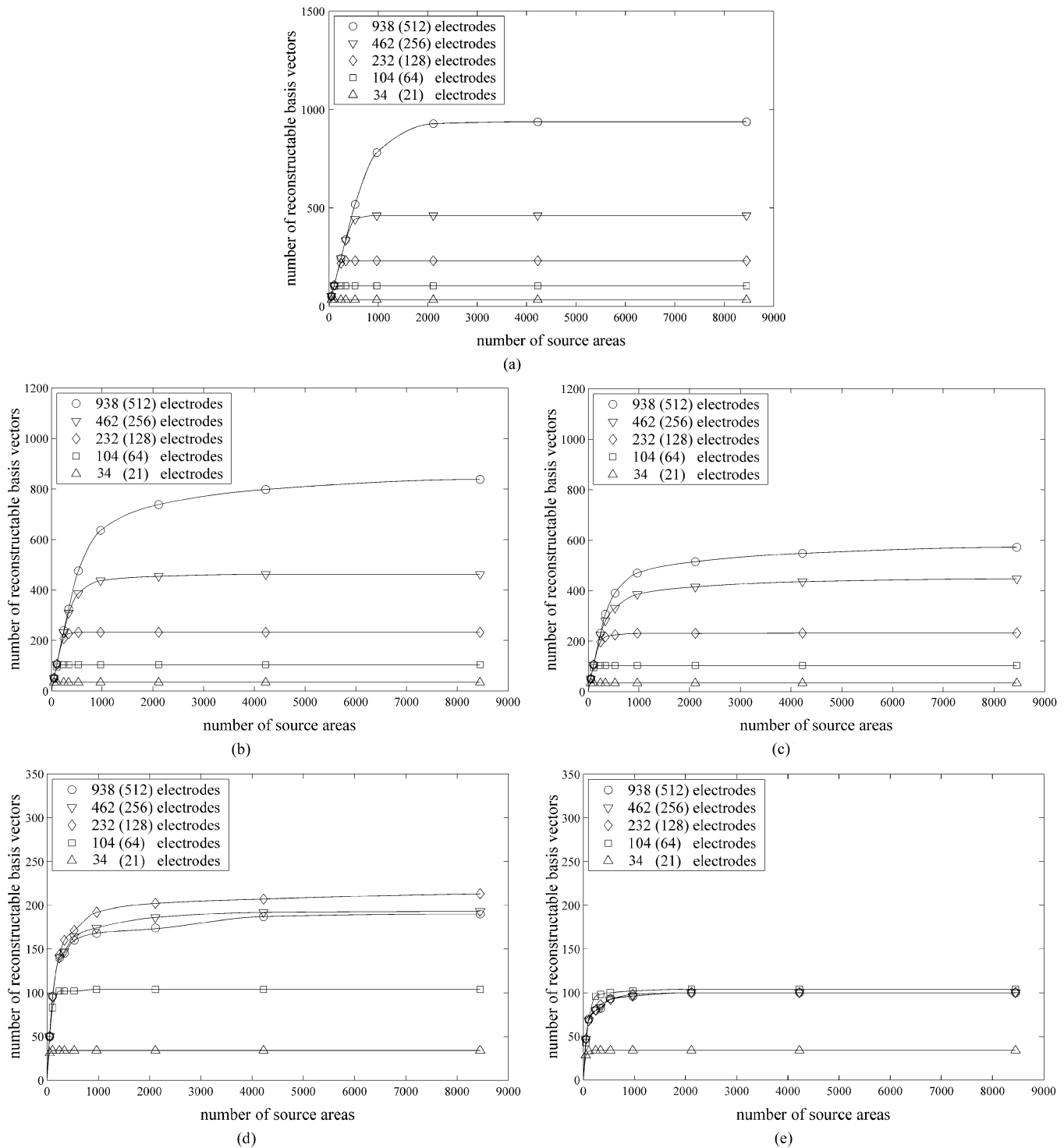


Fig. 3. Effect of the relative noise level on the number of basis vectors which can be reconstructed. The electrode numbers are given as they are placed on the entire spherical surface. The corresponding number of electrodes in the realistic system is given in brackets. As the measurement noise increases, the greater relative noise level value needs to be selected. It can be clearly seen that, as the measurement noise increases, the advantage of dense electrode systems over the use of sparser electrode systems is diminished. (d) and (e) correspond to realistic noisy measurement environments. The relative noise level in (c) is slightly better than in a realistic low noise environment. (a) NL = 0.001. (b) NL = 0.005. (c) NL = 0.01. (d) NL = 0.05. (e) NL = 0.1.

not increase. As the distance decreases from 50 mm, the HSV also decreases (spatial resolution improves).

Liu *et al.* [16] had a signal-to-noise ratio (SNR) of 10 in their simulations. This corresponds to a relative noise level of 0.1. Their results regarding the benefits of increasing the electrode number from 30 to 61 are in agreement with our results.

C. Measurement Noise

As already mentioned, with SVD both the errors in the measurement data (i.e., the measurement noise) and the error in the model (i.e., error in matrix A) can be taken into account when the accuracy of the inverse EEG problem is studied.

TABLE II
APPROXIMATED RELATIVE NOISE LEVEL RANGES IN WHICH SPECIFIC
ELECTRODE SYSTEMS ARE THE MOST OPTIMAL FOR OBTAINING THE BEST
POSSIBLE SPATIAL RESOLUTION

Realistic electrode system	Relative noise level range
512 electrodes	NL < 0.02
256 electrodes	0.02 < NL < 0.04
128 electrodes	0.04 < NL < 0.1
64 electrodes	0.1 < NL
21 electrodes	0.1 << NL

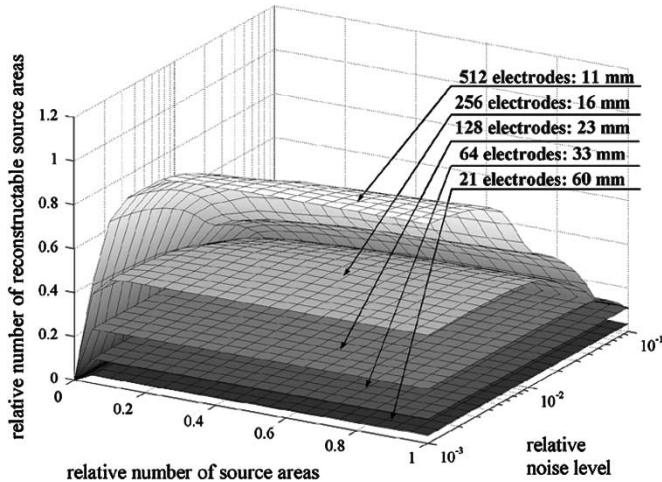


Fig. 4. Summary of the effect of the relative noise level and the number of electrodes and number of source areas on the solvability of the cortical potential distribution. As the noise ratio increases, the advantage of a dense electrode system over the use of sparser electrode systems is diminished.

The definition of measurement noise in our case, where the whole cortical potential distribution is to be solved, differs for example from the measurement noise in dipole localization. In dipole localization, the actual signal is the activity in the region of interest, i.e., the possible dipole location, and all other brain activity can be regarded as noise. When the whole cortical potential distribution is to be solved, the background activity of the brain is not considered to be noise but the actual signal. The noise in this situation is produced mainly by electrode contact noise and environmental noise.

Approximations of the electrode noise for normal biopotential electrodes vary between 1–15 μV [28] and 1–20 μV (rms) [29]. In our own experiments (unpublished results), we have obtained electrode noise values of 2–5 μV (rms) for wet gel Ag-AgCl electrodes of 10 mm in diameter. These agree well with the results in [29]. The EEG amplitudes vary typically between 10 and 100 μV [30]. In our estimations, we measured amplitudes between 15 and 85 μV (rms). Naturally, the amplitudes vary between different channels and the mean value of the amplitudes in our measurements was approximately 40 μV (rms). Using this information on the measurement noise and EEG signal levels, we may conclude that in a normal measurement environment the NL varies between 0.05 and 0.125. If the measurement noise could be reduced to 1 μV (rms), we could reach relative noise levels of 0.025. From this information and

referring to Table II, we may conclude that, in a normal measurement environment, the use of 128 or 64 electrodes is adequate to obtain the best possible spatial resolution. Thus, Fig. 3(d) and (e) corresponds to a normal measurement environment.

The results obtained by Lantz *et al.* [17] also support our findings. They conducted their study from EEG measured under clinical conditions and they found out that increasing the number of electrodes from 31 to 63 improved the source localization accuracy considerably and the increase to 123 electrodes improved the source localization accuracy. Their theoretical simulations also supported this finding. Even though they do not specify the SNR values during their clinical measurements, their results agree with ours obtained for clinical measurement environment.

If the measurement noise can be reduced to a very low level, the use of even 256 electrodes is indispensable to obtain the best possible spatial resolution. The relative noise level in Fig. 3(c) is slightly better than in a realistic low-noise measurement.

In the research of evoked potentials (EPs), there is a lot of background brain activity that is considered as noise. In EP research, averaging of several epochs is used to obtain a better SNR. The SNRs of averaged EPs are of the same magnitude as the relative noise levels estimated for solving the whole cortical potential distribution in this study.

In future studies, the effects of modeling errors should also be considered. The errors in matrix A include, for example, the modeling errors arising when a realistic computational model of the head is constructed. To estimate the error in matrix A , both the resolution and the accuracy of the calculation of the computational model can be taken into account. The inhomogeneous properties have the greatest effect on the accuracy of the computational head model. The correct anatomical shape and proper resistivity values of different tissues make a significant contribution to the accuracy of the model.

In the spherical head model, the largest modeling errors concern the skull. In the realistic head, the skull thickness varies considerably and the holes on it represent shunt paths for the currents. Furthermore, as has previously been shown, the ratio of the resistivities between different tissues has considerable effects on the spatial resolution of EEG [13], [14]. However, our results should be comparable to realistic geometry if the distance between the equivalent cortical surface and the scalp is close the distance used in this study.

Another source of uncertainty in matrix A is the accuracy of the electrode model. The electrodes can be modeled as point electrodes or realistical electrodes which take into account the electrode area and the shunting effect of the electrodes. The importance of the realistic electrode model increases concomitant with the number of electrodes [31]. Also, the correct localization of the electrodes affects the accuracy of the forward transfer matrix A .

These effects influencing the accuracy of matrix A should also be taken into account when defining the accuracy of the cortical potential distribution. However, here the limit of the maximal spatial resolution was obtained and the effect of increasing the measurement noise was studied, as the errors in A remained constant in all cases examined.

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